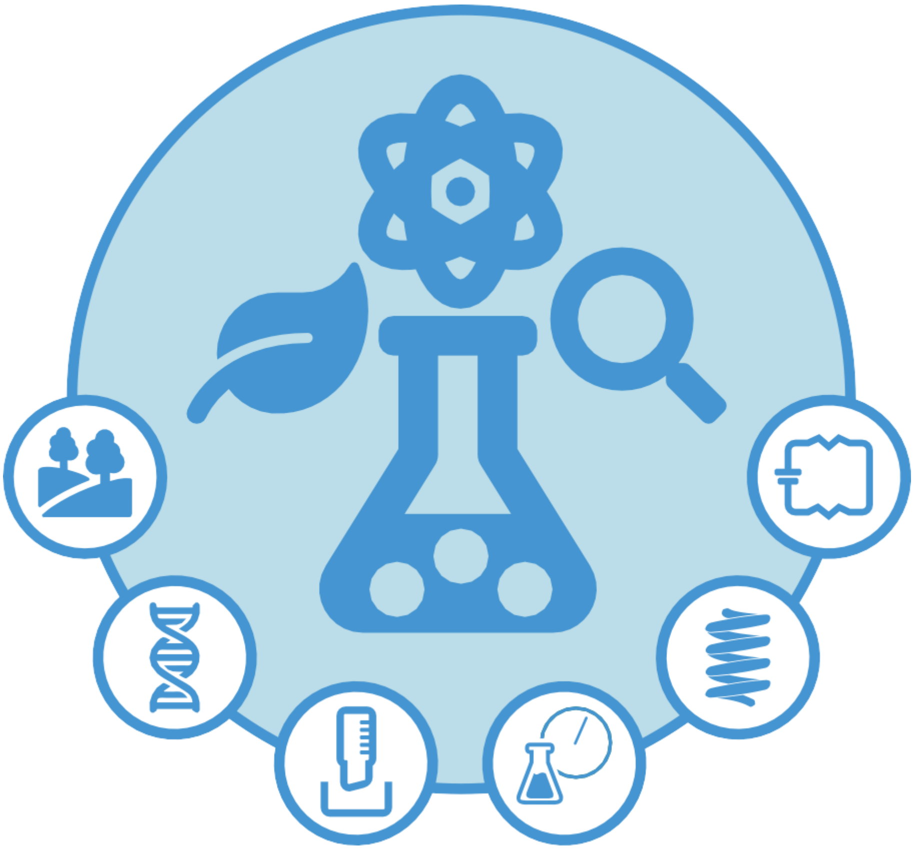
EXEMPLAR LESSON

# SPRING MASS SYSTEM 2: SIMPLE HARMONIC MOTION



# Acknowledgements

**Ministry of Education**

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For information on OpenSTEM Africa see: [www.open.edu/openlearncreate/OpenSTEM_Africa](www.open.edu/openlearncreate/OpenSTEM_Africa)

OPITO for their generous support, which has made OpenSTEM Africa and the development of the Virtual Laboratory and these materials possible.

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# Contents

# Exemplar lessons for the OpenSTEM Africa Virtual Laboratory applications

All the **exemplar lessons** are examples of lessons which could be used both individually and by whole classes of Senior High School (SHS) students in the elective sciences of Biology, Chemistry and Physics. Each of the lessons is linked specifically to one of the applications in the OpenSTEM Africa Virtual Laboratory. The exemplar lesson is created to give, both to SHS students and to SHS teachers, a clear example of the ways in which the applications can be used in the learning and teaching of practical science. There is a focus throughout the lesson on the student’s development of the practical and experimental skills which, along with knowledge and understanding, are integral to the profile of learning, teaching and assessment in SHS sciences.

The ‘you’ in this lesson is ‘you’, the Senior High School student. Remember that you can repeat the experiments and activities in this lesson as often as you have time for in class. This freedom to repeat experiments and activities is also important if you are accessing the lesson outside the classroom, for example for homework. Every application in the OpenSTEM Africa Virtual Laboratory contains real data – the experiments are real experiments. This means you might make mistakes the first or second or third time you try an experiment or an activity – and that is exactly what often happens in the real world in the sciences. So, it is helpful for you as a student to share in some of the real-world trial and error of science as you develop your skills as a scientist.

The exemplar lesson also contains a set of teaching notes at the end of this document for ‘you’ the SHS science teacher, to suggest how you might want to set up this particular lesson with one of your classes. Hopefully it will also generate ideas for other lessons on the same topic, or other lessons which use the same OpenSTEM Africa Virtual Laboratory application.

# Simple harmonic motion

This is the second of two lessons using the OpenSTEM Africa Spring mass system application. If you have not completed the first lesson (*Spring mass system 1: Determining a spring force constant*), please do so before starting this lesson.

# Lesson objectives

**By the end of the lesson, you will be able to:**

- Describe the harmonic properties of an oscillating spring.
- Explain how a period is converted to a frequency oscillation.
- Compare the effects of mass and displacement on the period of an oscillating spring.
- Calculate the spring force constant using mass and the square of the period.
- Describe how the stiffness of a spring relates the frequency of oscillation.

**The following practical and experimental Skills will be developed:**

- Observations
- Plotting
- Analysing
- Interpretation
- Reporting

## Background

In a previous lesson ‘Determining a spring force constant’ you learnt that as force is applied to a spring, it will extend in a linear and proportional manner until it reaches its **limit of proportionality** or **elastic limit**. You also learnt that **Hooke’s law** can be used to calculate the force constant of a spring – a measure of a spring’s stiffness within the limit of proportionality.

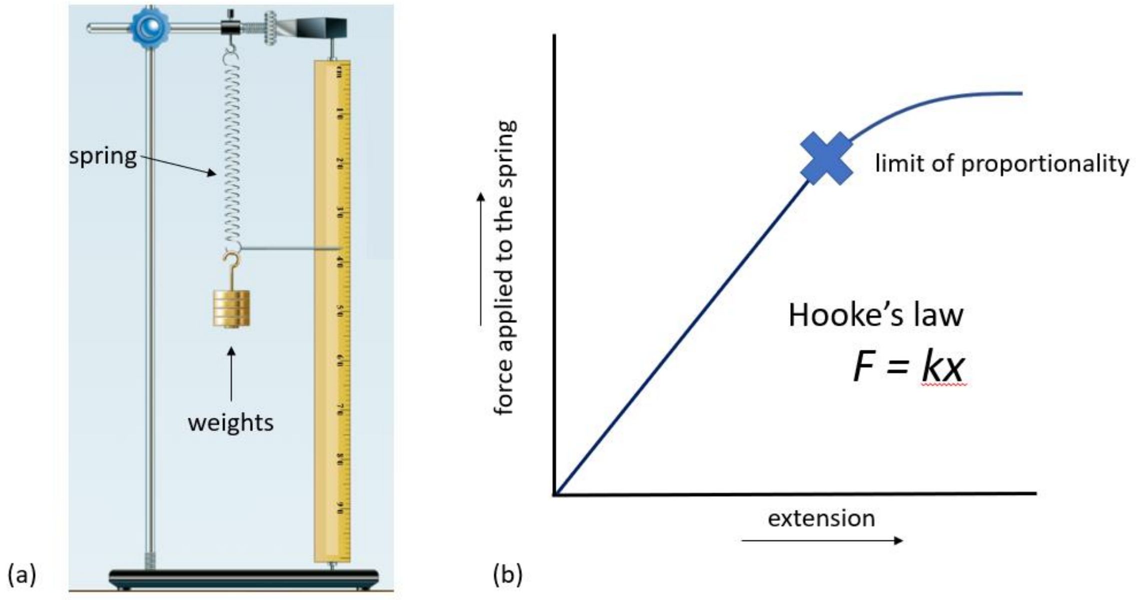


**Figure 1** (a) a spring mass system used to measure the extension of a spring when force is applied in the form of weights, (b) a graph showing the extension of a spring with increasing applied force. Hooke’s law is used to calculate the spring force constant (*k*), where *F* is the applied force and *x* the extension of the spring. Note: Hooke’s law only applies below the limit of proportionality when the extension of a spring is proportional to the applied force.

When a weight is added to a spring it will cause the spring to extend downwards to a new resting extension or **equilibrium point**, as shown in Figure 2.

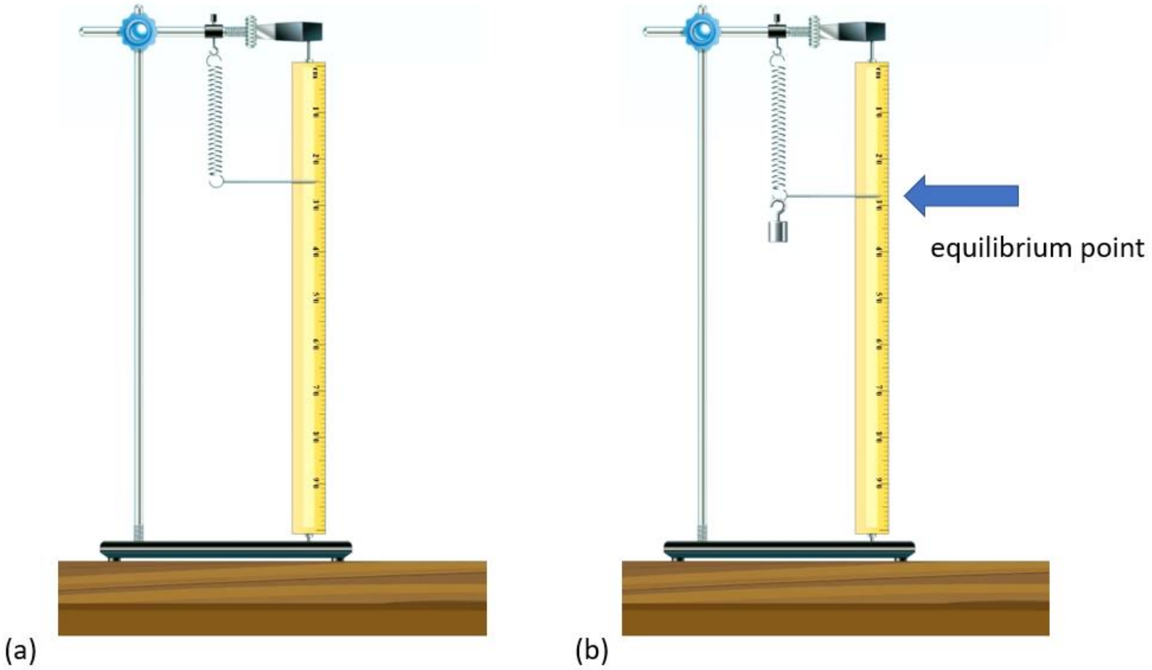


**Figure 2** (a) a spring at rest. (b) the same spring as in (a) but now extended to a new resting position by the addition of a weight, this new position is called the equilibrium point indicated by the blue arrow.

If the weight is pulled downwards (displaced) and then let go, the spring will start to oscillate around the equilibrium point (Figure 3).

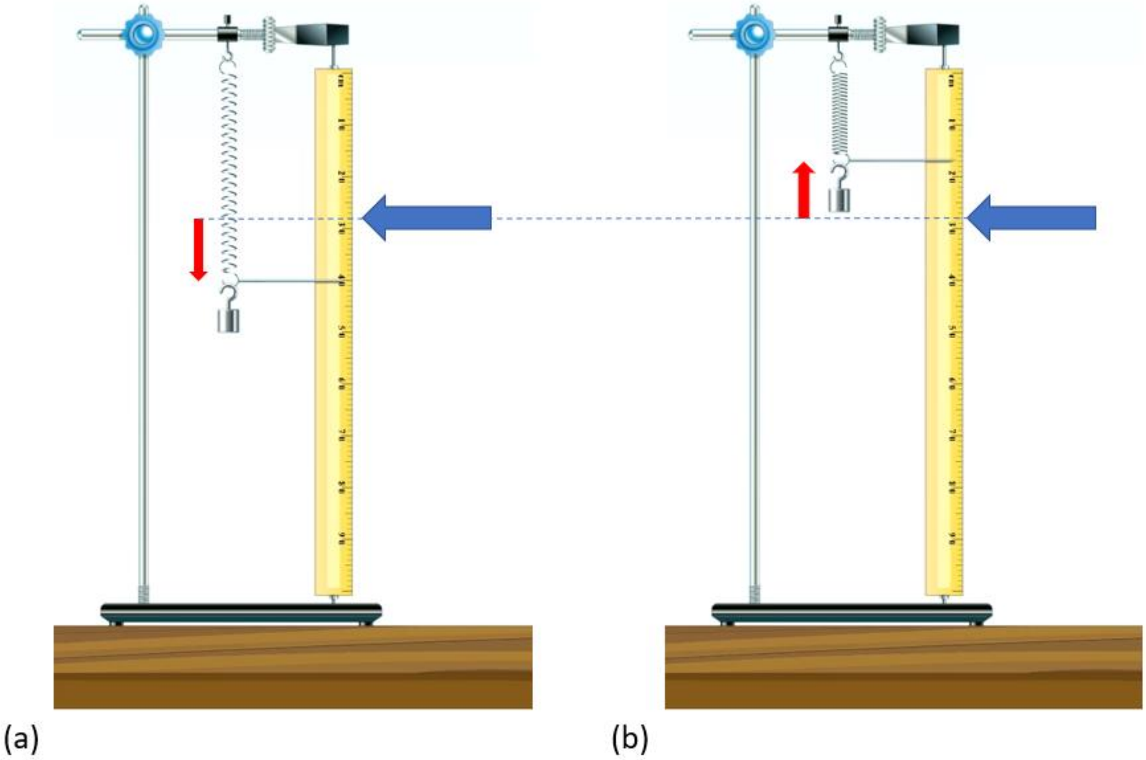


**Figure 3** (a) a weighted spring is pulled downwards (red arrow) away from its equilibrium point (indicated by the blue arrow and blue dashed line). (b) when the downward force is

released, the spring will move upwards, passing through the equilibrium point and reaching a point where the upward motion ceases (red arrow).

Figure 4 shows a plot of the motion of a weighted spring that was displaced by 10 cm and then released. As you can see, the weighted spring oscillates, moving up and down in a regular rhythm until it comes to rest – this type of oscillation is called **simple harmonic motion**. A defining feature of harmonic oscillation is that the rhythm is regular, that it is a series of repeated cycles occurring at a fixed **frequency**. The reason why a weighted spring oscillates is that the applied displacement initiates a complex interaction between kinetic energy and stored potential energy that sustains the oscillatory motion until it stops.

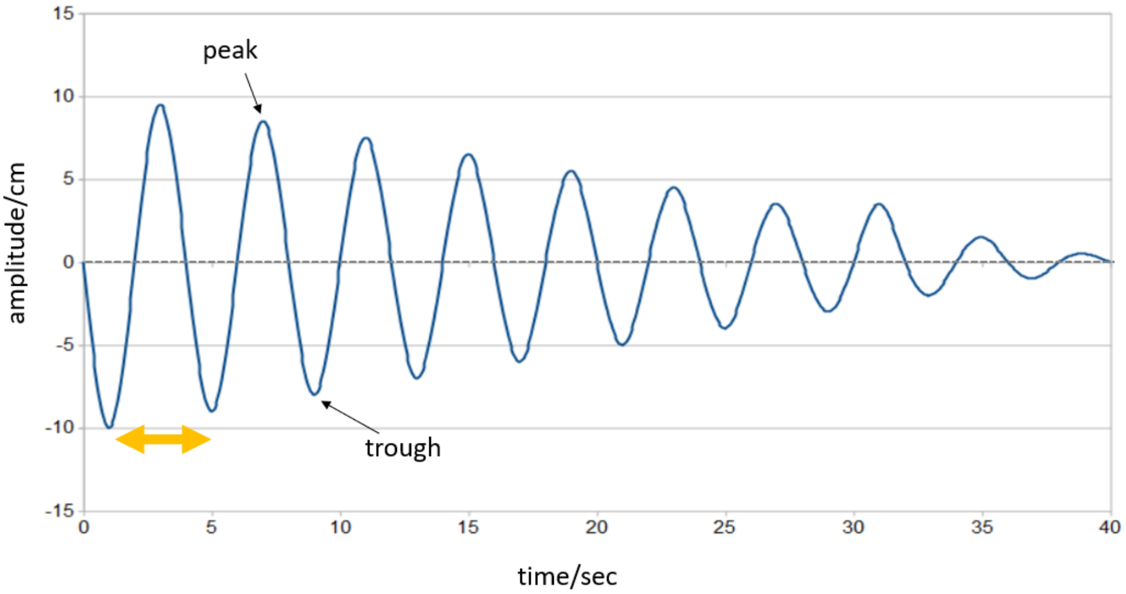


**Figure 4** The oscillatory motion of a weighted spring resembles a series of waves with peaks and troughs as it moves up and down around its equilibrium point (indicated by the dashed line). A full cycle (yellow double-headed arrow) is the movement of the spring as it starts from a starting point, for example the trough, moves upwards to the peak and then returns to its starting point. In the example shown, the cycle takes about four seconds.

The example of a harmonic oscillation shown in Figure 4 has a slow cycle (taking about four seconds) and because a cycle is measured using units of time it is called the **period** (*T*). You will also notice that the oscillations do not continue indefinitely, they eventually stop – this is because of friction, both from the air and the microstructure of the spring itself.

| **If it takes an oscillating spring 5 ms to move from its equilibrium point to the peak of an oscillation, what is the period?** |
|---|
| Go to Appendix 3 for the answer. |

The period of an oscillation can also be expressed as a frequency. The unit used for frequency is **hertz** (Hz). A frequency of 1 Hz is equal to a period of 1 second, or in other words, one cycle per second. An oscillating spring with a period of 500 ms will complete two cycles in 1 second and in this case the frequency is expressed as 2 Hz (two cycles per second).

| **If an oscillating spring has a period of 20 ms, what is its frequency of oscillation?** |
|---|
| Go to Appendix 3 for the answer. |

In the practical activity you will explore the relationship between mass, the displacement of a spring and the period. You will also use the period and mass to measure the spring force constant.

# Practical activity

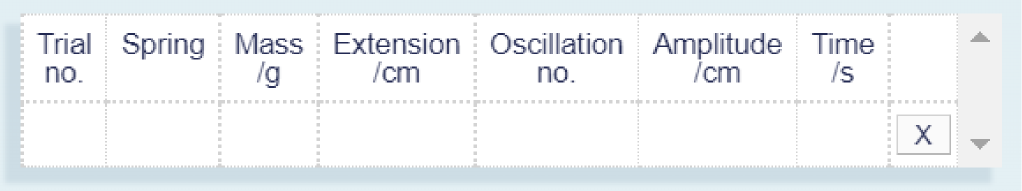
In lesson 1 *Determining a spring force constant* you discovered that the spring force constant of a spring can be calculated by measuring the extension of the spring after the addition of a weight and then applying Hooke’s law. In this activity you will use the ‘Simple harmonic oscillation’ option located in the Spring mass system application to examine the dynamic properties of an oscillating spring and use your observations to calculate the spring force constant using the period of oscillation and the weight (mass) added to the spring. However, your first task is to examine the relationship between the weight added to a spring, the displacement (amplitude) used to initiate the oscillation and the period of the oscillation.

Before you access the Spring mass system application homepage, first read the following instructions.

## Task 1

Your first task is to explore the relationship between the weight (mass) added to a spring, the displacement applied to a weighted spring (the amplitude) to initiate harmonic oscillation, and the period of oscillation.

Copy the following table headings into your laboratory notebook. You will use this to record the data generated by the spring mass system.



| Trial no. | Spring | Mass /g | Extension /cm | Oscillation no. | Amplitude /cm | Time /s | |
|---|---|---|---|---|---|---|---|
| | | | | | | | X |

**Figure 5** The table headings used for the investigation of simple harmonic oscillations using the spring mass system application.

When you access the experimental homepage (*do not do this until you have read the instructions thoroughly*), select the ‘Simple harmonic oscillation’ option and enter the experiment.

You will have the choice of two springs (a soft and a stiff spring). Once you have selected a spring, use the sliding bar to select and add a weight to the spring and then wait for the spring to settle at its new equilibrium point.

Once the spring has settled, use the cursor (together with the left click function on a PC mouse) to drag the weight downwards, extending the spring and then release the weight (by releasing the left click button). The weighted spring will now oscillate until it comes to rest at its equilibrium point. You have three options for measuring the period of oscillation. The one that closely resembles a laboratory is the ‘Unlimited’ option. If selected, the spring will oscillate until it comes to a stop, and you will need to use a stopwatch to calculate the period. The other two options will stop the spring oscillating after 10 and 20 cycles respectively. For the purposes of Task 1, select the ‘10’ oscillations option. Remember to divide the time recorded by the number of oscillations to obtain the period (the time it takes to complete a full cycle).

Now you are ready to collect your data. For your chosen weight and spring, apply a series of different displacements (amplitudes) and record your observations.

Look at your recorded observations and discuss the relationship between mass, amplitude and period with your classmates.

## Task 2

In this task you will use the mass and period to calculate the spring force constant of your chosen spring. For this experiment, select a spring and then starting with the 25 g weight, apply a series of amplitudes and record your settings and the periods in your laboratory notebook. Now repeat this process using the 50 g weight and then the 100 g weight.

Remember, if you use the ‘10’ or ‘20’ oscillations option, you will need to divide the recorded time by the number of oscillations to obtain the period.

The following equation can be used to calculate the period if the mass and spring force constant are known:

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Here, the period ($T$) is measured in seconds (s), the mass ($m$) in kilograms (kg) and the force constant ($k$) in **newtons** per metre (N/m).

This equation can be arranged so that the force constant can be measured using the slope of a graph.

This is achieved by first squaring the period (so $T$ becomes $T^2$).

$$T^2 = 4\pi^2\left(\frac{m}{k}\right)$$

And then rearranging the equation further so mass ($m$) becomes a separate quantity.

$$T^2 = \left(\frac{4\pi^2}{k}\right)m$$

Now a plot of $T^2$ against mass will have a slope that approximates the quantity

$$\left(\frac{4\pi^2}{k}\right)$$

And can be rewritten so slope = $\frac{4\pi^2}{k}$, and from this a simple rearrangement will give the equation for the force constant ($k$).

$$k = 4\pi^2/\text{slope}$$

Using your observation, plot the square of the period (s$^2$) against the mass (kg) and use the slope to calculate the spring force constant (Figure 6).

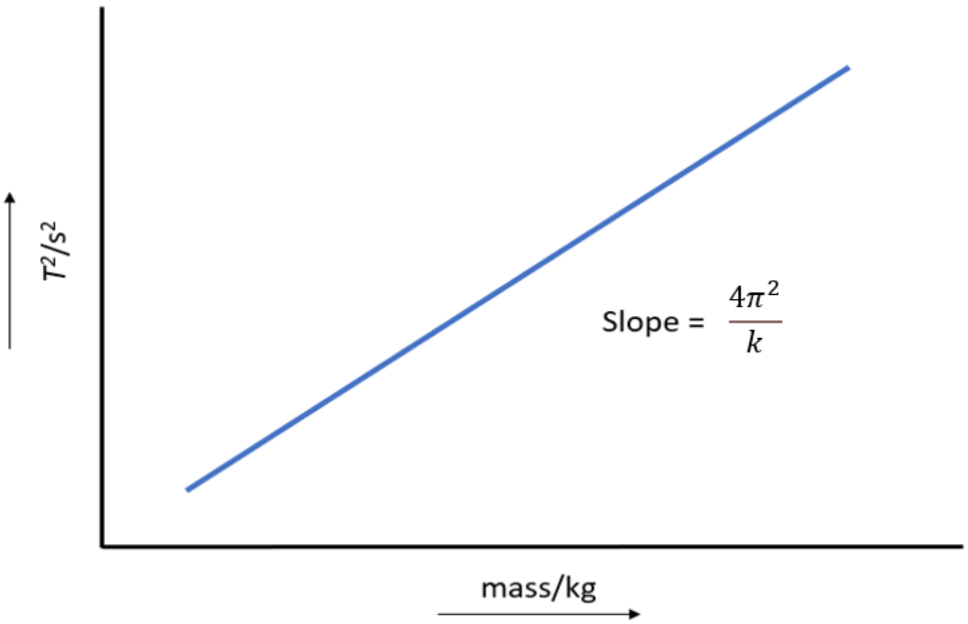


**Figure 6** The spring force constant can be calculated using the slope of the square of the period against mass.

Now that you have read these instructions you are ready to enter the experiment.

**Spring mass system**

Go to the OpenSTEM Africa Virtual Laboratory.

Click on the icon to access the Spring mass system application homepage.

Watch the introductory video before entering the experiment.

## Summary

Using the Spring mass system application, you have explored the properties of simple harmonic motion. You have examined how the mass of a weight and the degree of applied displacement (amplitude) affect the period of oscillation of a spring. You have also seen how the square of the period and mass can be used to calculate the spring force constant.

For further explanation of some of the terms used in this lesson, go to the **Glossary**.

# Quiz

Answer the questions, then search for the correct answers in Appendix 4.

## Question 1

**What is the period of a weighted spring oscillating at a frequency of 7 Hz?**

a) 14 minutes
b) 1.4 seconds
c) 0.14 seconds (correct)
d) 0.014 seconds

## Question 2

**Which one of the following statements is correct?**

a) The size of the amplitude used to initiate oscillation determines the period.
b) The mass of the weight added to a spring determines the period, such that larger the mass the shorter the period.
c) The mass of the weight added to a spring determines the period, such that the larger the mass the longer the period. (correct)
d) The period of an oscillating spring is not dependent on the mass of the weight added to the spring.
e) Smaller amplitudes applied to a weighted spring will increase the length of the period.

## Question 3

**What is the period for this spring under the following conditions?**

An oscillating spring has an added mass of 4 kg and a spring force constant of 10 N/m.

Write your answer, rounded to decimal places, below.

______________

## Question 4

**Is the following statement true or false?**

The period of an oscillating spring is determined by the added mass (*m*) and the spring force constant (*k*) as shown in the equation for determining the period (*T*) in Task 2. From this equation is can be concluded that the stiffer the spring, the shorter the period.

## Question 5

A plot of period squared (T2) against mass (m) has a slope of 0.356. What is the spring force constant for the spring used to collect the data for this plot? (π has been rounded to 3.14)

a) 110.8 N/m (correct)
b) 11.1 N/m
c) 1.1 N/m
d) 1108 N/m

# Glossary

**Elastic limit** – The point at which a spring has been stretched/deformed beyond its ability to return to its original shape and length – the spring has been damaged.

**Equilibrium point** – The resting extension of a weighted spring. When a spring undergoing simple harmonic motion will oscillate around its equilibrium point.

**Frequency** – The number of cycles (oscillations) that occur in one second.

**Hertz** – The unit (Hz) used to express frequency.

**Hooke's law** – $F = kx$ where $F$ is the applied force in newtons (N), $k$ is the spring force constant and $x$ is the extension of the spring measured in metres (m). A spring force constant has the units of N/m. Hooke's law only holds below the limit of proportionality.

**Limit of proportionality –** The point at which the relationship between force and extension is no longer linear and proportional.

**Newton –** A measure of force due to the acceleration of gravity. At sea level (where mass = weight), a 1 kg mass has a force of 9.8 newtons (N).

**Period** – The period ($T$) is time taken for an oscillation to complete a full cycle.

**Simple harmonic motion –** Oscillatory motion of a spring around its equilibrium point.

# Appendix 1: Teacher notes – organisation of the lesson

Teaching notes for the Spring mass system and the exemplar lesson on simple harmonic motion.

This lesson, using the spring mass system, links to the following units in the Teaching Syllabus for Physics:

- SHS 2 Section 2 Mechanics, Unit 3 Oscillatory motion
- SHS 2 Section 4 Waves, Unit 2 Wave motion

Ideas for organising this exemplar lesson link directly to activities and teaching examples in the OpenSTEM Africa CPD units on Using ICT to support learning, and Approaches to active notetaking.

A full list of the OpenSTEM Africa CPD units can be found at: https://www.open.edu/openlearncreate/CPD_units

## Overview

If possible, this lesson should take place in the ICT Lab in your school if this can be arranged through your Head of Science and the Head of ICT. If the lesson takes place in the ICT Lab, it may be possible for each student to work individually at a computer; otherwise divide the class so that students are in small groups at a computer.

If it is not possible to use the ICT Lab for this lesson, then try to set up this lesson in your classroom. You may be lucky enough in your school to have a set of 'empty' tablets or mobile phones which students can use. Or you may be able to bring into the classroom a laptop connected to the internet or to your school intranet – and perhaps connected to a projector to make it possible for the whole class to view at once. If access to ICT is a real challenge in your school but you want your students to view an experiment, you might be able demonstrate it to small groups of your students at a time, using your own mobile phone

Whatever way(s) you set up the class, it would still be helpful to the students to be able to work in pairs or small groups for at least some of the lesson. Do remember as well that students need desk space to be able to write in their notebooks and to draw tables and diagrams.

## Steps in organising the lesson

**Step 1:** This takes place at the beginning of the lesson where you and your class access the OpenSTEM Africa Virtual Laboratory spring mass application. Have students work in pairs to pre-read the Background section of the exemplar lesson and ask each other the questions in the Background section. While they are doing so, you may want to walk round the class and check their laboratory notebooks, as accurate note-taking and filling in the tables is important for this exemplar lesson.

**Step 2:** Check students' understanding by asking them the questions in the Background section. Have each person in the pair create the tables in their own laboratory notebook in preparation for their data collection.

**Step 3:** Once the students have seen the video one time, and if it is helpful to do so, give the class a set time to draw spring mass system in their laboratory notebook and label it. Within each pair, have them check each other's work.

**Step 4:** Make sure that each pair has access to/can see the computer screen to begin the actual experiments. Ensure that each pair knows how to carry out the experiments– or if you are using a laptop/projector, that you draw on the expertise of the class as you go through each step of the spring mass system experiments – i.e., ask them what the next step is

**Step 5:** Have the class follow the instructions for each of the spring mass experiments. Make sure, if working in a pair on a PC, that each student in the pair gets to follow all the steps; if working in a group on a PC, have the group leader ensure that everyone in the group is involved.

**Step 5:** What they write in their tables will be agreed between the pair or within the group but allow enough time for everyone in the class to fill in their own set of tables. Have them check each other's writing.

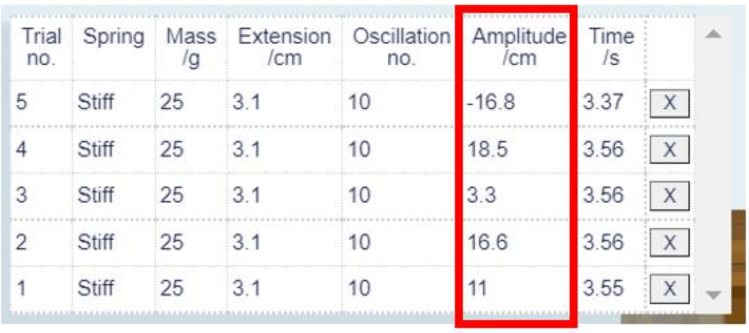
**Step 6:** Ten minutes before the end of the lesson, tell the students to complete the quiz.

# Appendix 2: Teacher notes – out of the lesson

## Task 1

Students were tasked with exploring the relationship between mass, the amplitude used to displace a weighted spring with the period of oscillation using the ‘Simple harmonic oscillation’ experiment accessed via the Spring mass system homepage.

Below is a table of observations collected using the stiff spring.



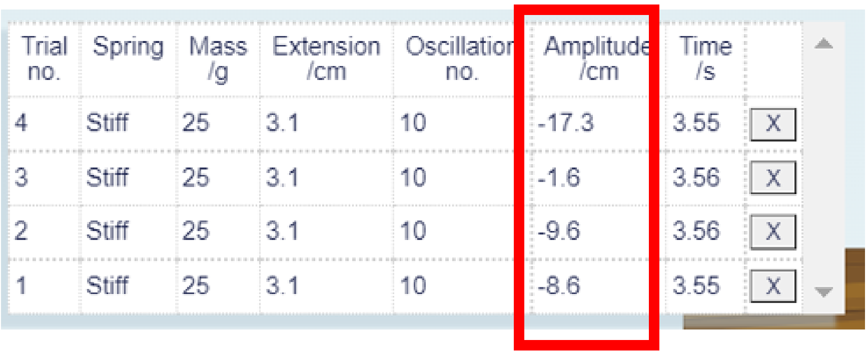
| Trial no. | Spring | Mass /g | Extension /cm | Oscillation no. | Amplitude /cm | Time /s | |
|---|---|---|---|---|---|---|---|
| 5 | Stiff | 25 | 3.1 | 10 | -16.8 | 3.37 | X |
| 4 | Stiff | 25 | 3.1 | 10 | 18.5 | 3.56 | X |
| 3 | Stiff | 25 | 3.1 | 10 | 3.3 | 3.56 | X |
| 2 | Stiff | 25 | 3.1 | 10 | 16.6 | 3.56 | X |
| 1 | Stiff | 25 | 3.1 | 10 | 11 | 3.55 | X |

**Table 1** Data collected using the stiff spring and downward displacement.

Please note that these data were collected using the **‘10’ oscillation option**, meaning that the **Time/s** column has recorded the time it takes for 10 oscillations to occur (so the period of one oscillation is a tenth of the time shown in the table).

The key piece of learning is that the period is independent of the amplitude used to initiate the oscillations. *Note:* all the observations obtained in Table 1 were collected by pulling the spring downwards. You will see that *Trial no. 5* has a negative value, and the period is smaller – the negative value indicates that the trial is invalid, this is because the downward pull was a large one and the weighted spring hit the clamp, thus compromising the experiment.

The data set shown in Table 2 were collected using an upward displacement, and here the negative sign has been used to indicate this.



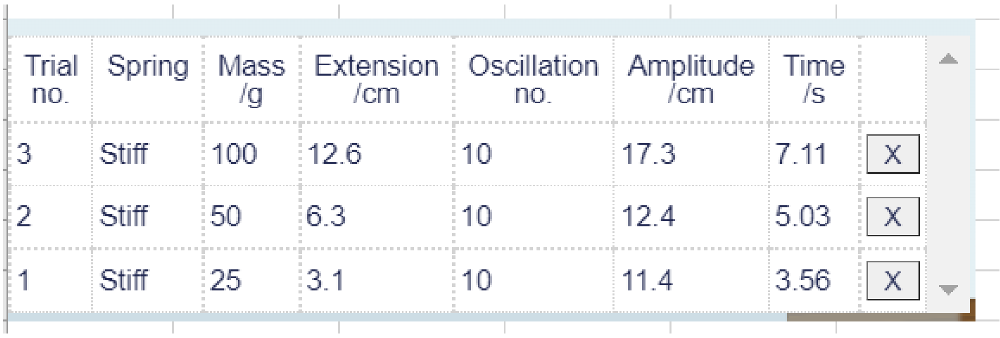
| Trial no. | Spring | Mass /g | Extension /cm | Oscillation no. | Amplitude /cm | Time /s | |
|---|---|---|---|---|---|---|---|
| 4 | Stiff | 25 | 3.1 | 10 | -17.3 | 3.55 | X |
| 3 | Stiff | 25 | 3.1 | 10 | -1.6 | 3.56 | X |
| 2 | Stiff | 25 | 3.1 | 10 | -9.6 | 3.56 | X |
| 1 | Stiff | 25 | 3.1 | 10 | -8.6 | 3.55 | X |

**Table 2** Data collected using the stiff spring and a series of upward displacements.

## Task 2

In Task 1 the students discovered that the period of oscillation is independent of the size of the displacement amplitude. In Task 2 they then explored the effect of increasing mass on the period and how both the mass and period can be used to calculate the force constant.

Table 3 shows the effect of increasing mass on the period of oscillation for the stiff spring.



| Trial no. | Spring | Mass /g | Extension /cm | Oscillation no. | Amplitude /cm | Time /s | |
|---|---|---|---|---|---|---|---|
| 3 | Stiff | 100 | 12.6 | 10 | 17.3 | 7.11 | X |
| 2 | Stiff | 50 | 6.3 | 10 | 12.4 | 5.03 | X |
| 1 | Stiff | 25 | 3.1 | 10 | 11.4 | 3.56 | X |

**Table 3** The effect of increasing mass on the period of oscillation.

The key piece of learning here is that the period increases with the mass added to the spring.

The observations in Table 3 can be used to calculate the spring force constant. This is done by plotting the square of the period against mass and using the slope of the graph to calculate the force constant. However, before this can be done the data in Table 3 needs to be transformed, so the period is expressed as $T^2$ and the mass in kg (Table 4).

| **mass (kg)** | **T (s)** | **$T^2$ ($s^2$)** |
|---:|---:|---:|
| 0.025 | 0.356 | 0.126736 |
| 0.05 | 0.503 | 0.253009 |
| 0.1 | 0.711 | 0.505521 |

**Table 4** Transformed data from Table 3.

The transformed data in Table 4 is plotted in Figure 7.

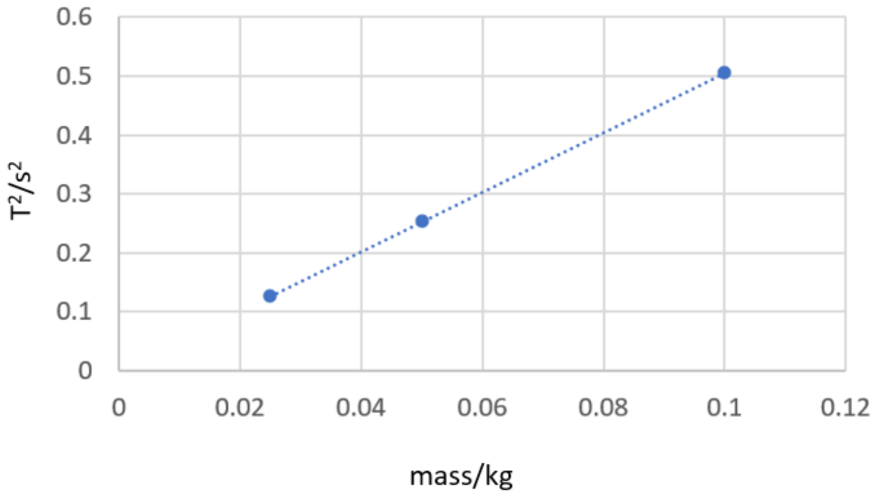


**Figure 7** The square of the period for the stiff spring is plotted against the mass. The slope has a value of approximately 5.05.

The slope of the graph shown in Figure 7 has a value of 5.05. This can now be used to calculate the force constant of the stiff spring by using the following equation:

$$k = 4\pi^2/\mathrm{slope} = 39.44/5.05 = \mathbf{7.81\ N/m}$$

The equation $T = 2\pi\sqrt{\frac{m}{k}}$ can be used to predict how changing mass or the stiffness of a spring will affect the period of oscillation. An increase in mass (*m*) will lengthen the period whereas an increase in stiffness (*k*) will decrease it. The latter can be demonstrated by comparing the slopes of the stiff and soft springs on the same graph.

# Appendix 3: In-text question answers

**If it takes an oscillating spring 5 ms to move from its equilibrium point to the peak of an oscillation, what is the period?**

**Answer:**
Remember, the period is the time it takes for an oscillation to complete a cycle. If the starting point is the equilibrium point (EP), then the path taken to complete a cycle is: (i) EP to the peak, (ii) peak to the EP, (iii) EP to the trough, (iv) trough to the EP. Therefore, the period is 20 ms (4 x 5 ms).

**If an oscillating spring has a period of 20 ms, what is its frequency of oscillation?**

**Answer:**
An oscillating spring with a period of 20 ms will complete 50 cycles in one second. It will have a frequency of 50 Hz.

# Appendix 4: Quiz answers

Correct answers are **highlighted in green**.

## Question 1

**What is the period of a weighted spring oscillating at a frequency of 7 Hz?**

a) 14 minutes  
b) 1.4 seconds  
**c) 0.14 seconds**  
d) 0.014 seconds

### *Feedback*

7 Hz means that the spring has completed 7 full cycles in one second. To convert frequency into period, divide 1 second by 7.

## Question 2

**Which one of the following statements is correct?**

a) The size of the amplitude used to initiate oscillation determines the period.  
b) The mass of the weight added to a spring determines the period, such that larger the mass the shorter the period.  
**c) The mass of the weight added to a spring determines the period, such that the larger the mass the longer the period.**  
d) The period of an oscillating spring is not dependent on the mass of the weight added to the spring.  
e) Smaller amplitudes applied to a weighted spring will increase the length of the period.

### *Feedback*

Look again at the equation used to calculate the period – from this it can be determined that the larger the mass the longer the period.

## Question 3

**What is the period for this spring under the following conditions?**

An oscillating spring has an added mass of 4 kg and a spring force constant of 10 N/m.

Write your answer, rounded to decimal places, below.

____**3.97**____seconds

### *Feedback*

The period for a spring with a force constant of 10 N/m and an added mass of 4 kg is 3.97 seconds (for this calculation π has been rounded to 3.14).

## Question 4

**Is the following statement true or false?**

The period of an oscillating spring is determined by the added mass (*m*) and the spring force constant (*k*) as shown in the equation for determining the period (*T*) in Task 2. From this equation is can be concluded that the stiffer the spring, the shorter the period.

**Answer: True**

### *Feedback*

Stiffer springs have larger force constants than softer springs. From the equation for calculation of *T* in Task 2, the larger the force constant, the shorter the period. From the same equation, it is also true that the larger the mass the longer the period.

## Question 5

A plot of period squared (T2) against mass (m) has a slope of 0.356. What is the spring force constant for the spring used to collect the data for this plot? (π has been rounded to 3.14)

- **a) 110.8 N/m**
- b) 11.1 N/m
- c) 1.1 N/m
- d) 1108 N/m

### *Feedback*

The spring force constant (*k*) can be calculated using the following equation:

$$k = 4\pi^2/\text{slope}$$

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